JOINT INSTITUTE FOR AERONAUTICS AND ACOUSTICS

National Aeronautics and Space Administration

Ames Research Center

JIAA TR - 80

Stanford University

SOME THOUGHTS ON DELTA WING VORTEX BREAKDOWN

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SEPTEMBER 1987

ABSTRACT

An assessment of the fluid dynamics associated with delta wing vortex breakdown has been conducted. A preliminary numerical experiment, consisting of the numerical solution of the three-dimensional thin-layer Navier-Stokes equations, was carried out. The objective of the numerical experiment was to evaluate the ability of numerical simulations to describe the complex structures arising in delta wing vortex breakdown within reasonable computational cost.

Some insight into the delta wing vortex breakdown phenomenon is also obtained from an overview of the approaches and theories applied to the much simpler case of the breakdown of an axisymmetric vortex. Current theories and computations for this case are discussed.

A numerical experiment on an almost conical vortex flow, representative of the vortices on the leeward side of delta wings, is proposed for further study.

INTRODUCTION

The notion of vortex breakdown refers to the drastic changes that take place in a vortex whose axis is aligned with an external main stream, changes that usually occur when the vortex is exposed to an adverse pressure gradient. Breakdown is characterized by a sudden swelling of the vortex core, which starts in a fairly symmetrical fashion, accompanied by an equally sudden decrement of the vortex centerline velocity, which in many cases leads to localized stagnation and recirculation. The swelling of the vortex core is usually followed by a fairly coherent form of wave motion, generally non-axisymmetric in form, and then by intense large scale turbulence.

This description of vortex breakdown corresponds to the version of the phenomenon which is most relevant to the aerodynamics of slender bodies, such as delta wings and missiles, at moderate to high angles of attack.

Delta wing vortex breakdown was first reported by Peckham[1], and was observed to cause significant changes in the wing aerodynamic coefficients. A critical survey of the vortex breakdown phenomenon has been reported by Leibovich[2], who conducted a critique of present theories. An overview of vortex breakdown as a fundamental fluid-mechanical problem was reported by Escudier[3].

Even in its simplest form, the vortex breakdown problem has challenged analytical treatment. In the case of a delta wing vortex, which constitutes the problem of greatest practical importance, the formulation of a purely theoretical treatment would appear exceedingly difficult. In this work a preliminary computation of delta wing vortex breakdown will be carried out, followed by a discussion of the theoretical explanations applicable to an axisymmetric vortex, in a manner that a better understanding of the delta wing case will be facilitated.

In a classical experiment reported by Lambourne et al.[4], the delta wing vortex breakdown phenomenon was classified in two types: B-type, also called bubble breakdown, and S-type, referred to as spiral breakdown.

In the absence of breakdown, the vortex core of the vortical structure on the leeward side of a delta wing has a jet-like axial velocity distribution. At the onset of breakdown the axial velocity undergoes a sudden deceleration, usually leading to stagnation or flow reversal at the axis. This process occurs over a distance of one to two core diameters. After breakdown of either type has occurred the vortex acquires a wake-like character, with greater unsteadiness and a thickened core. In the case of a vortex in a tube, spiral breakdown causes a thickening of the core which is about 30% greater than in the case of bubble breakdown. The factor determining which of the two forms of breakdown occurs appears to be the swirl number, defined as $\Omega = \Gamma/(U_\infty d)$, with Γ the vortex circulation measured at the edge of the viscous core, U_∞ the free-stream velocity, and d the core diameter. For a fixed Reynolds number, Sarpkaya[5] and Faler et al.[6] demonstrated that breakdown makes a transition from the S-type to the B-type as the swirl number is increased. This transition occurs rather sharply, suggesting the existence of a critical swirl number. Although the position of breakdown is affected by the Reynolds number, this fact is not necessarily to be interpreted as an indication that breakdown itself is a viscous phenomenon. Rather, an increment in the Reynolds number causes a thinner vortex core, thereby altering the swirl number. All indication suggests that vortex breakdown itself is an intrinsically inviscid phenomenon[7]. In this regard, Escudier et al.[8] managed to correlate the breakdown position to the non-dimensional group $Re\Omega^3$. Most of these observations have been made through experiments with vortices in tubes, a set-up which achieves greater controllability and simplicity than a delta wing. However, they are expected to carry over to the case of delta wings. The upstream movement of the breakdown and the transition between the two types have been documented to occur in delta wings in much the same way as in vortices in tubes[9]. Breakdown for delta wings is usually of the spiral form, which may be attributed to the swirl number not being high enough, in most practical aerodynamic situations, to sustain a bubble-type breakdown.

Numerical studies of vortex breakdown have been carried out both for axisymmetric configurations and delta wings. The axisymmetric configurations usually dealt with assume that a vortex in a viscous flow evolves in a cylindrical tube. Steady state numerical solutions of the Navier-Stokes equations are sought, typically in steady-state form[10,11,12]. Krause[13] utilized a combination of boundary layer equations for the flow field ahead of breakdown, and the complete, time-dependent Navier-Stokes equations for the flow field at breakdown and behind. All of these attempts make the significant assumption that vortex breakdown is described by a steady-state solution of the Navier-Stokes equations. In fact, there is no strong evidence that this is the case. The effect of boundary conditions and grid size on the steady state solutions of reported calculations has not been fully investigated. In the case of inviscid simulations [12], a periodic structure is observed, which can be strongly affected by the nature of the boundary conditions. In Krause's computation [13], the boundary layer assumptions ahead of the breakdown inhibit the communication with the upstream boundary conditions, thereby making the steady state solutions less reliable.

Fully three-dimensional, steady-state Navier-Stokes computations on a strake-delta wing have been conducted by Fujii et al.[14], who detected spiral-type vortex breakdown and found good agreement with experiments. In this short work this approach is explored further with the aim of assessing the potential of three-dimensional Navier-Stokes computations for describing vortex breakdown at moderate computational cost.

ASSESSMENT OF CFD DESCRIPTION OF DELTA WING VORTEX BREAKDOWN

In view of the extreme difficulty posed by analytical approaches to the vortex breakdown problem in the case of axisymmetric support flows - "support flow" is the flow field that would exist in the absence of breakdown -, the problem for conical support flows, such as is the case in delta wings, is likely to be elucidated by the application of CFD.

A preliminary evaluation of the potential of CFD to describe the vortex breakdown phenomenon in delta wings was undertaken. A three-dimensional, thin-layer Navier-Stokes (TLNS) code was applied to a triangular delta wing with a 40 deg apex angle.

These preliminary numerical experiments were aimed at assessing the following.

a) Suitability of the TLNS code to the description of delta wing vortex breakdown with reasonable cost on the Cray XMP supercomputer. Here two aspects are to be considered: the degree of mesh refinement required to capture breakdown, and the limitations imposed by a steady-state calculation. Both these aspects exert an impact on the cost of the effort. Previous calculations have shown the onset of breakdown to be in good agreement with experiments for the case of a double delta wing [14]. Such calculations made use of a mesh with about 8×10^5 grid points, which puts it outside the range of the Cray XMP due to computational cost. Grid densities four to five times lower would be needed to satisfactorily run the calculations on the Cray XMP, corresponding to one to two hours of CPU time. The code used here can be operated in pseudo-time or in time accurate form. Time accuracy probably increases the cost of calculation by about a factor of four.

b) Suitability of the graphics and flow field interpretation software available at NASA Ames to visually examine the burst flow field. Much of the understanding of vortex breakdown will be derived from careful analysis of a pictorial representation of the flow field.

c) The first steps in designing a numerical experiment to explore the effect of adverse pressure gradients on almost-conical vortices: It is commonly accepted that vortex breakdown in delta wings is caused by the onset of an adverse pressure gradient, arising due to the presence of a trailing edge. In actual flight the characteristics of this pressure gradient are connected to the wing shape and angle of attack. It is felt that better understanding of the breakdown phenomenon can be gained by controlling the adverse pressure gradient as an independent parameter.

The cross-section of the delta wing used to conduct this assessment consisted of two parallel sides and semi-circular leading edges. The cross-sectional shape varied conically between the apex and 70% of the chord, with a relative thickness of 1.8%. The cross-sectional shape between the 70% chord location and the trailing edge had a linearly decreasing thickness, ending in a sharp trailing edge.

Experimental evidence has indicated that the separated flow on a wing of these characteristics would exhibit onset of bursting (the crossing of the trailing edge by the burst region) at an angle of attack of about 32.5 deg. At an angle of attack of 40 deg. this wing is expected to exhibit a fully burst vortex over a major part of its chord.

The computational domain consisted of a circular tube surrounding the wing, with a diameter twice the length of the root chord. The downstream boundary was placed two root-chord lengths aft of the trailing edge. The upstream boundary was located one root-chord length ahead of the apex. The boundary conditions consisted of free-stream values on the forward and lateral walls of the domain, and first order extrapolations on the aft wall. Two grid densities were assessed: a coarse grid, consisting of about 50000 grid points, and a fine grid, with about 120000 grid points. Mach number was specified at 0.3 and Reynolds number at 1×10^6. Coarse grid calculations required about 10 minutes of Cray XMP CPU time for converged solutions, while fine grid calculations required about 55 minutes. In both cases only pseudo-time solutions were carried out, with about 500 iterations required for convergence.

Fig. 1 shows a view of a longitudinal cut through the flow field approximately following the vortex core, for an angle of attack of 32.5 deg. The vortex bursting indicated by a region of flow reversal is clearly seen. Experimental information would suggest that at this angle of attack the breakdown region is expected to appear close to the trailing edge. However, there is considerable scatter in the determination of the angle of attack for the onset of breakdown. This fact, coupled with the thinness of the present wing, would suggest that the description of Fig. 1 is quite plausible.

A similar view for the case of angle of attack of 40 deg. is shown in Fig. 2. A well-developed vortex breakdown can be seen, giving indication of the presence of periodic structures. Experimental evidence indicates that such an extent of breakdown is to be expected for this angle of attack.

Fig. 3 shows the same view from a computation with the coarse grid. The extension of the vortex bursting, however, is much smaller than would be expected for this value of the angle of attack.

Careful observation of Fig. 2 indicates the presence of two structures. The burst region extends over two-thirds of the wing chord, and it expands in a rather conical fashion after its onset. One periodic structure is observed near the wing surface, evidenced by the vortex-like velocity fields on the cutting plane. Another structure can be seen above the previous one, close to the upper edge of the burst region. An interpretation of these structures is best accomplished by releasing particle tracers in the proximity of the onset of breakdown. In general, a trial and error process is required to locate the tracer. With help from longitudinal and transversal flow field cuts, a successful tracer was released, whose trajectory is illustrated in Figs. 4 and 7. The tracer describes the edge of the viscous core, clearly illustrating the onset of a spiral wave. Eventually the tracer leaves the core, migrating outside the field of interest. It is quite likely that this migration is not physical, but the result of inaccuracies in the integration process required to recreate the particle paths. The structures nearest the wing surface, seen in Fig. 3, correspond to cuts through the twisting vortex core. The weakest structures farther out from the wing surface are likely to represent secondary vortices produced by the shear resulting from the interaction of the flow induced by the twisting vortex core and the free-stream. A much more detailed study of these structures would be required to characterize them more precisely. The coiling of the main vortex in these structures has been observed experimentally by Werle[9].

It can be stated that vortex breakdown is triggered by an adverse pressure gradient along the vortex core, which in the case of a delta wing is facilitated by the presence of a trailing edge. It would be desirable to isolate the effect of an adverse pressure gradient from the wing configuration and angle of attack. A numerical experiment to achieve this goal would consist in artificially producing an adverse pressure gradient along an almost conical vortex. This can be achieved by enclosing a thin conical delta wing, set at a moderate angle of attack, in a circular tube with permeable walls, as illustrated in Fig.6. By prescribing an outflow velocity along the tube walls, an effective decrement of the velocity of the flow that would exist in the absence of the wing would be obtained, thus generating the desired adverse pressure gradient. If the wing is sufficiently slender, the adverse pressure gradient imposed by the outflow wall boundary conditions of the tube would dominate any local changes due to the wing itself.

DISCUSSION

A plausible explanation for vortex breakdown is that a "sudden" transition between an upstream and a downstream state occurs, in a manner conceptually analogous to the hydraulic jump[15]. A brief explanation of the hydraulic jump is useful in interpreting the line of reasoning that will follow.

A plane layer of fluid in a given configuration of a Poiseuille-type motion, subjected to gravity, may admit a second configuration, compatible with conservation of mass and momentum, depending upon the so-called criticallity condition of the first configuration. If the first configuration is such that the speed of the fastest upstream-travelling infinitesimal wave vanishes, the flow field is called critical. If such speed is greater than zero, the flow field is called supercritical. It is easy to show that a second configuration corresponding to a supercritical flow is achieved with dissipation of kinetic energy. This means that a second configuration is possible, its occurrence depending on the downstream boundary conditions. The transition from one configuration to the other requires a mechanism for energy dissipation. In the hydraulic jump problem this mechanism is twofold: kinetic energy is dissipated through turbulence into heat, and translational kinetic energy can be transferred to, and entertained in, the energy associated with large-scale unsteadiness. The second configuration is subcritical and wave motion is always possible. The jump will occur if the upstream flow is supercritical and the downstream boundary conditions are incompatible with that supercritical flow. Where the transition* will occur will depend on the ability of the energy-dissipating mechanisms to facilitate the transition to the subcritical state meet the downstream boundary conditions. The details of the transition can be very complex.

The vortex bursting phenomenon can be characterized in a similar way. To a given supercritical swirling flow there corresponds a subcritical state. At what level of "criticallity" the transition occurs and through what means is at the heart of understanding vortex bursting. If vortex bursting is framed in this way, it becomes integral part of the explanation that the bursting occurs to satisfy downstream boundary conditions. In aeronautical applications, these conditions are increased pressure near the trailing edges of the aircraft wings.

The notion of finite transitions was introduced by Benjamin[16] and has since been put on firm mathematical footing. Benjamin's analysis of transitions is based on allowing for a loss of total flow force, and is limited to ducts of constant cross section.

Although the concept of transition between different states as the underlying cause of vortex breakdown is not without criticism, as will be discussed below in more detail, it is extremely likely that the true nature of the phenomenon is closely associated with such a concept. This is supported by the finding that, after breakdown, the axial velocity profile changes in a way consistent with Benjamins prediction [6]. If the notion of transition is accepted, the discussion then centers around the mechanisms that bring it about. The case of a vortex in a cylindrical tube is best suited for visualization and understanding.

Transitions can be explored in an ad-hoc way by specifying a velocity profile, characterized by a small number of parameters, and then seeking multiple solutions to the conservation equations. In this process, momentum is usually conserved and energy is allowed to dissipate. In the course of this work it was found that transitions obtained in this way depend crucially on the type of velocity and swirl profiles used to approximate the vortex. In many instances, no multiplicity of solutions was detected. For this reason the concept of transitions has limited value when applied to obtain quantitative information on bursting.

Experiments on vortices in tubes indicate that the first change a vortex undergoes as it begins to burst ("begins" is meant here to suggest a process in space, not in time) consists of a well defined structure, which takes the form of either a symmetric expansion of the core, or of a spiraling contortion of the core. The bursting is accordingly classified as "bubble" type or "spiral" type, in a way analogous to delta wing breakdown. Experiments also show that in most cases the motion that follows is usually disorganized, with the vortex core appearing significantly swollen and unsteady. Carefully controlled experiments, however, permit a process of unbursting to occur, in both kinds of bursting. The two types of bursting in tubes described here are the most relevant ones. Experiments in tubes have shown that several variations of these basic types are possible[6], leading to a wider classification in up to five bursting types.

Consistent with the concept of transitions, the experimental evidence would suggest that the onset of a bubble somewhere along the core (or the beginning of a spiral distortion of the core) is the beginning of a process that will transfer energy from the steady motion into unsteady motion, thus eventually leading to the second state. Four questions arise at this point: What is the bubble (or spiral)? why does it establish itself at a given position along the vortex axis? what does the onset of such bubble or spiral have to do with criticality? and finally, why and how is the steady-unsteady energy transfer process caused by their presence? These will be answered sequentially.

The nature of the bubble (or spiral) is best described by a solitary wave or soliton. Solitons of both types have been shown to be solutions to the differential equation governing small departures of fluid particles from their steady-state positions in a vortical support flow (the "unburst" vortex.) The discussion will now be specialized to the case of bubbles, for which comparison between theory and experiment has been more extensively and reliably made. There are two main reasons why it is logical to assume that the bubbles seen in experiments are solitons. One is the similarity between the two flow fields, as substantiated by experimental[17] and computational[18] mappings of the bubble flow field. The calculated soliton constitutes a bubble which looks quite similar to the observed ones[19]. Both, aspect ratio of the bubbles and axial velocity behave much the same way in the two cases. The second, and more powerful argument, however, has to do with criticality. It has been shown mathematically[20] that a soliton, which in general can move either upstream or downstream the vortex core, will be at rest with respect to a stationary observer in a vortex flow which is slightly supercritical. How supercritical will depend on the characteristics of the soliton, and presumably on the type of vortical flow[20]. It can also be shown that a reliable measure of criticality is given by the maximum swirl angle of the flow, which can also be measured with accuracy (approximately two degrees[20]). If the experimental bubble were a soliton, the swirl angle for which it appears should be the same that would correspond to the level of criticality that would make the soliton stationary with respect to an observer at rest. This is indeed found to be the case to experimental accuracy.

Pursuing the nature of the bubble further, both experiments and CFD computations have also shown that the first bubble is followed by other, usually smaller bubbles[10,11]. Three explanations can be explored: the observed bubble is a window into an essentially periodic phenomenon, which in an ideal case would extend infinitely far upstream and downstream the vortex core. Although the most recent CFD computations show a pattern of bubbles which would appear to support this view, it will be argued below that their appearance is conditioned by the computational method itself (through the imposition of periodic boundary condition on a finite domain). The above view, on the other hand, assumes that the phenomenon would be periodic in a domain of infinite extent. This quite apart from the conflict that could arise with the soliton interpretation for the "first" bubble, which is nicely supported by experiments through the criticality argument already discussed.

A second explanation for the "secondary" bubbles is that they are remnants of a train wave which arose because the soliton did not manage to accomplish the transition in one stroke (Benjamin's view). This view has merit, but must be rethought in terms of realistic transitions that preserve momentum, as opposed to Benjamin's transitions. A third explanation is that the smaller bubbles are part of a dispersive tail of solitary waves, which will be ordered sequentially by decreasing amplitude. This view has the strength that the unsteady solutions of the Kortweg-de Vries equation, (the equation governing the first soliton in the absence of, or with very little, dissipation), do constitute a dispersive tail[20] (Dispersive meaning that all of them, except the main soliton, will, after some time, be washed downstream and disappear.) This explanation is very appealing, and although in most numerical experiments the bubbles are not seen to disappear, very recent work appears to indicate that no steady state can be found. It could then be suspected that many CFD solutions may be forcing a transient picture to appear as steady, most likely through the method of solution. This will be elaborated on below.

The fourth question is physically the most significant one, why and how is the steady-unsteady energy transfer process triggered by the presence of the soliton, or bubble. The qualification steady-unsteady should rather be referred to as non-wave vs. wave, since the very presence of waves, even if they are steady with respect to a stationary observer, implies that energy has been diverted from the support flow (this would fit with Benjamin's view of the little bubbles as a wave train.) The energy transfer process occurs because without it there would no transition and the flow may be bound to meet incompatible downstream boundary conditions. Once the soliton (assumed for the discussion to be of the bubble type) has established itself somewhere along the core, the core fluid attempts to negotiate turning around it, a process which leads, through viscosity, to the formation of a wake. In general, the process of negotiating the soliton will also lead to the formation of axisymmetric waves, with which energy is associated. From then on the process is open to accommodate a number of disturbances, certainly in a very complex fashion, with the eventual result that transition will have occurred "on time". After an axisymmetric disturbance sets in, energy can be transferred to spiral disturbances through the interaction of non-linear wave modes, with energy going from axisymmetric to spiral modes[20]. After the spiral and the axisymmetric modes begin to interact, the flow becomes disorganized, perhaps aided by some of those modes having become unstable, and eventually large-scale turbulence together with some large scale spiral structures take over the energy engaging process. The soliton will adjust its size and region of criticality such that the completion of the process guarantees that boundary conditions are met.

The concept of transition implies that either momentum or energy is dissipated. Although transitions preserving momentum and energy can be shown to exist, both mathematically[22] and experimentally[23], such flow configurations have regions in the core where the total head differs from the free-stream value by the free-stream dynamic pressure. In experiments this situation is simulated by injecting air into the core of a vortex in water. It is hard to see how this fact would be relevant to the phenomenon as seen on delta wings. In experiments conducted in tubes with diverging walls, the loss of momentum occurs through forces acting on the walls of the tube. The energy loss is accounted for by heat dissipation or unsteady motion. In the case of a delta wing, both mechanisms are possible. Momentum loss can be associated with changes in wing lift and hence in drag.

CONCLUSIONS

An assessment of the vortex breakdown problem as posed by delta wings was conducted. A preliminary evaluation of the application a three-dimensional thin-layer Navier-Stokes code to the description of the vortex breakdown phenomenon in delta wings also undertaken.

It is concluded that important contributions to the understanding of vortex breakdown as occurring in delta wings are more likely to be derived from carefully conducted numerical experiments than from the elaboration of mathematical theories, due to the extreme difficulty of the analytical nature of the problem.

A simple numerical experiment attempting to explore the effect of pressure gradient on breakdown, such that the pressure gradient is controlled independently from the wing attitude and configuration, was proposed as a suitable area of numerical experimentation.

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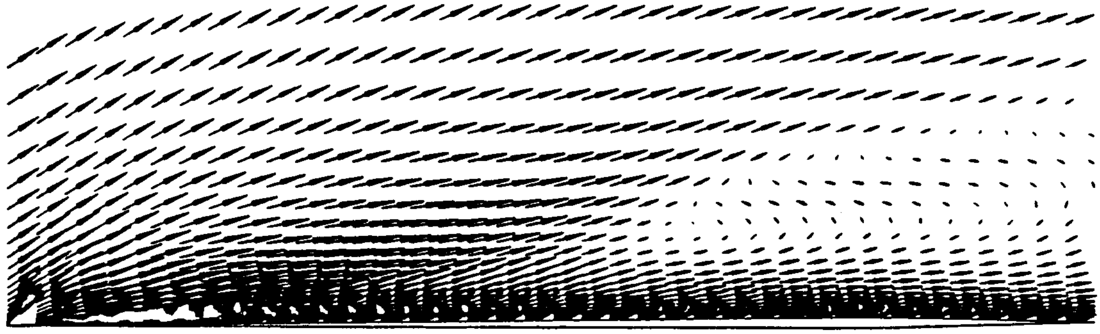

Fig. 1 Lateral view of vortex breakdown, apex angle 40 deg, angle of attack 32.5 deg, fine grid.

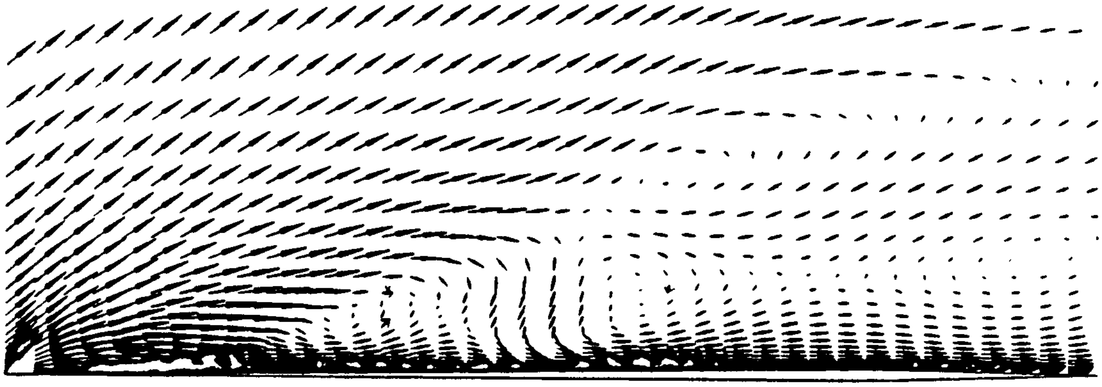

Fig. 2 Lateral view of vortex breakdown, apex angle 40 deg, angle of attack 40 deg, fine grid.

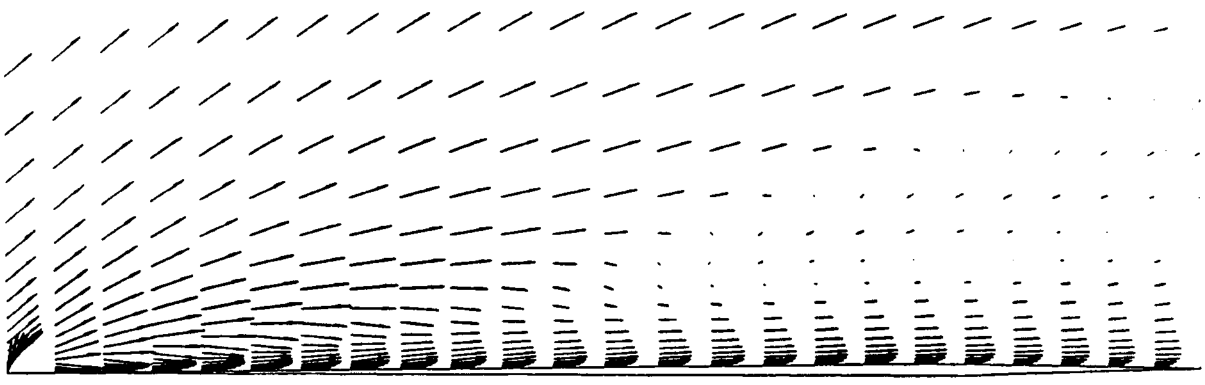

Fig. 3 Lateral view of vortex breakdown, apex angle 40 deg, angle of attack 40 deg, coarse grid.

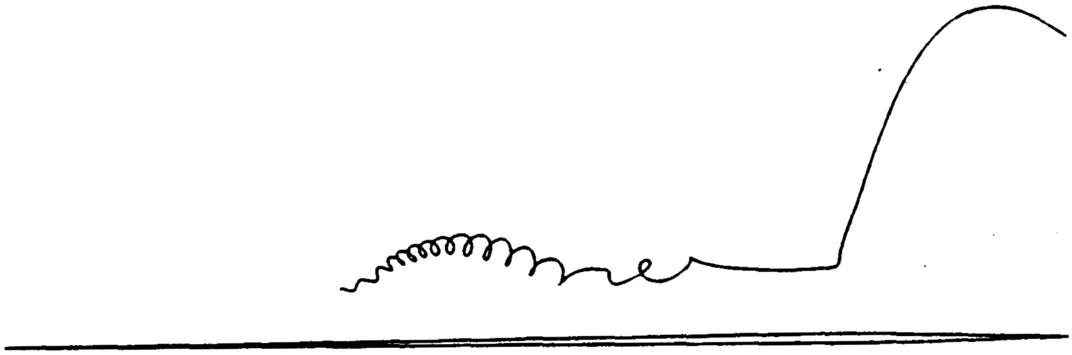

Fig. 4 Vortex core tracing in breakdown field of Fig. 2, side view.

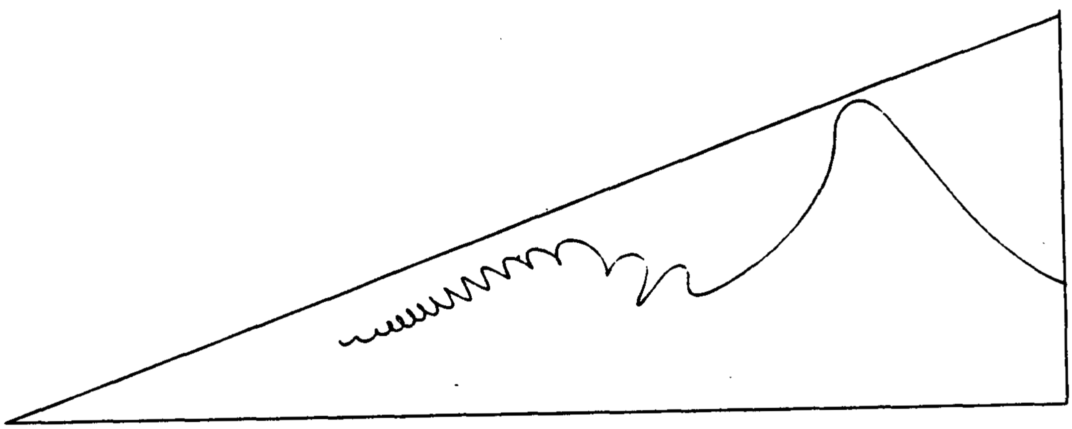

Fig. 5 Vortex core tracing in breakdown field of Fig. 2, top view.

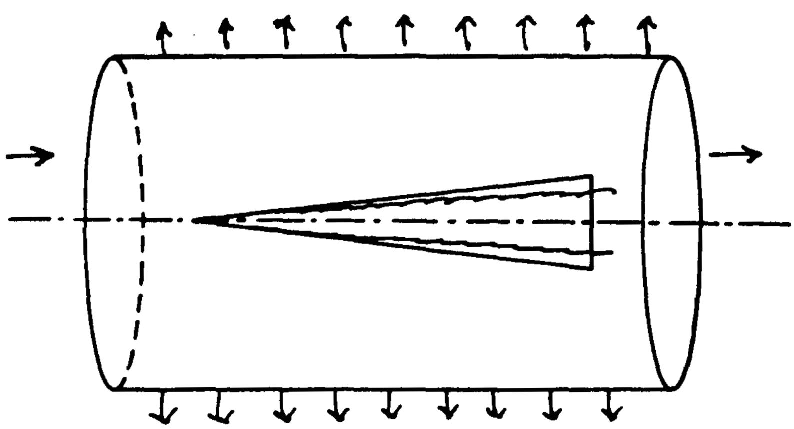

Fig. 6 Numerical experiment set-up to explore adverse pressure gradient effect.